Session Materials:

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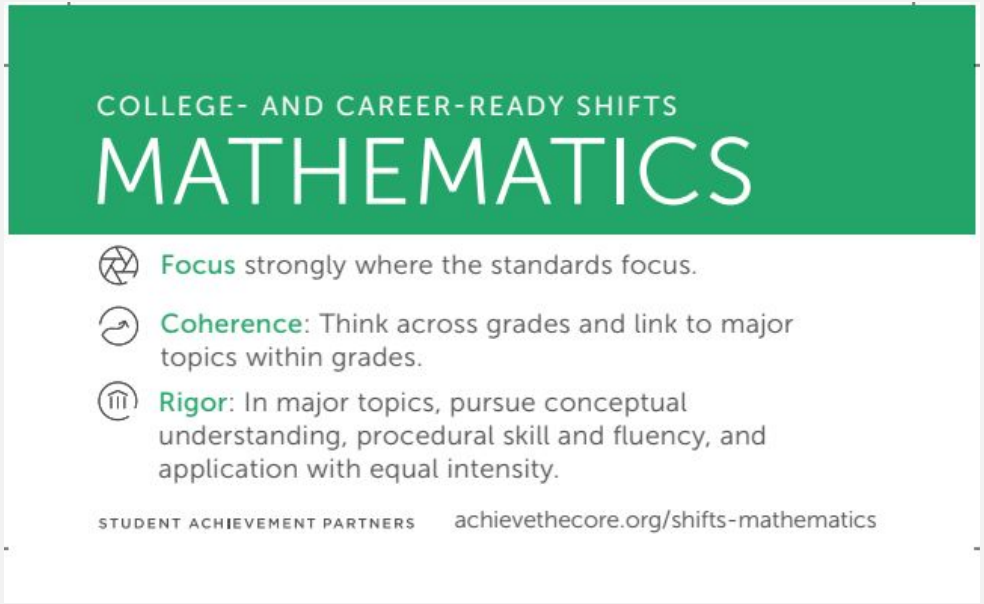

Upping the Rigor of your Math Assessments

Assessment Conference
January 15,2019

OFFICE OF PUBLIC INSTRUCTION
ELSIE ARNTZEN, STATE SUPERINTENDENT
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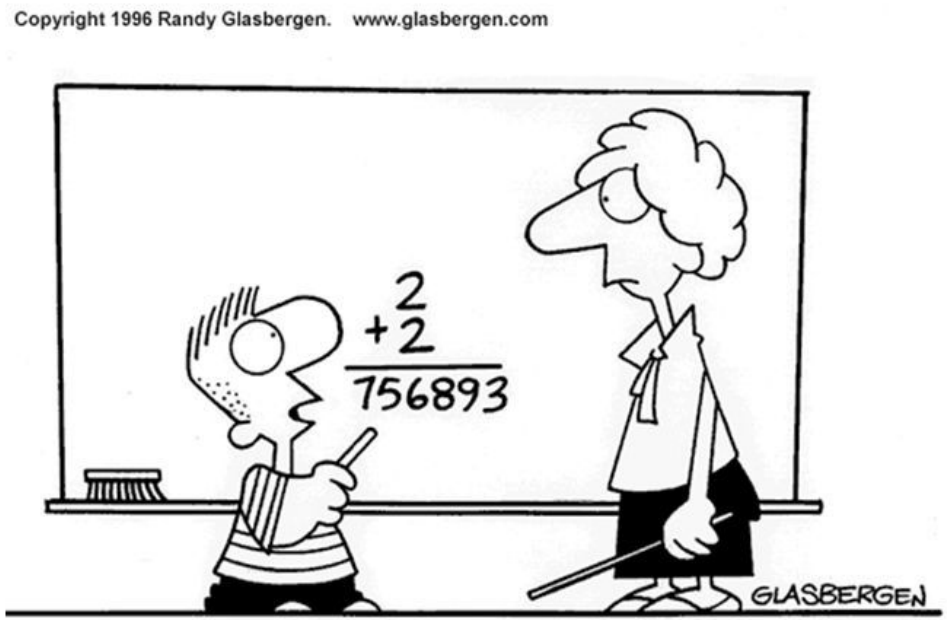
Putting Montana Students First

Rigor



"In an increasingly complex world, sometimes old questions require new answers."

True or False

1. Lots of homework is a sign of rigor
2. Rigor means doing more
3. Rigor is not for everyone
4. Providing support means lessening rigor
5. Resources do not equal rigor

True or False

1. Lots of homework is a sign of rigor FALSE
2. Rigor means doing more FALSE
3. Rigor is not for everyone FALSE
4. Providing support means lessening rigor FALSE
5. Resources do not equal rigor TRUE

The Five Sources of Cognitive Complexity

1. Content: Claims + Targets + Standards
2. Practices: Scaffolding
3. Stimulus Material: Item Stimuli
4. Response Mode: Item Type
5. Processing Demand: Depth Of Knowledge (DOK)

Learner-Centered Design: A Cognitive View of Managing Complexity in Product, information and environmental design. .

By Wayne Reeves

Definitions

Claims: are a broad statements that will outline the outcomes achieved with mastery of the standards within it.

Targets: Targets further clarify the knowledge and specific skills that cross over a cluster of standards. [Cluster Titles]

Standards: What students should know and be able to do.

#1 Content: CLAIMS

Claim #1— Concepts & Procedures Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency.
Claim #2— Problem Solving Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies.
Claim #3— Communicating Reasoning Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.
Claim #4— Modeling and Data Analysis Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.

Targets

Targets: Highlights

- Bullets: Standards

GRADE 3 STANDARDS

Operations and Algebraic Thinking (OA)

Represent and solve problems involving multiplication and division.

- Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as 5 × 7. (3.OA.1)
- Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8. (3.OA.2)
- Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (3.OA.3)
- Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = ? ÷ 3, 6 × 6 = ?. (3.OA.4)

Understand properties of multiplication and the relationship between multiplication and division.

- Apply properties of operations as strategies to multiply and divide. Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.) [Students need not use formal terms for these properties.] (3.OA.5)
- Understand division as an unknown factor problem. For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8. (3.OA.6)

Multiply and divide within 100.

- fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers. (3.OA.7)

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

- Solve two-step word problems using the four operations within cultural contexts, including those of Montana American Indians. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. [This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order.] (3.OA.8)
- Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that four times a number is always even, and explain why four times a number can be decomposed into two equal addends. (3.OA.9)

Content: Targets + Standards

Claim	Target	Standard
1: Concepts and Procedures: Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.	**A:** Represent and solve problems involving multiplication and division.	**3.OA.1** Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as 5 × 7. EQ **3.OA.2** Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8. **3.OA.3** Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. **3.OA.4** Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = ___ ÷ 3, 6 × 6 = ?.

Content: Targets + Standards

1. Check out the Claims, Targets, Standards alignment documents from Santa Rosa City Schools
2. Go to: **goo.gl/GnfqwG**
3. Claims Targets Standards document
4. What do you notice? What do you wonder about the alignment?

#3 Stimulus Material: Item Stimuli

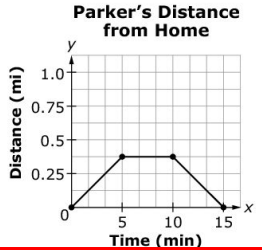
Kaden has 7 bags of animal toys. Each bag has these animal toys in it.
- 1 whale toy
- 5 dolphin toys
- 2 turtle toys

How many animal toys does Kaden have altogether?

Select **all** of the equations that show how to find the total number, t, of animal toys.

- ☐ $7 \times 8 = t$
- ☐ $7 + 1 + 5 + 2 = t$
- ☐ $7 \times (1 + 5 + 2) = t$
- ☐ $7 + (1 \times 5 \times 2) = t$

The graph shows Parker's distance from home over time.



Based on the graph, determine whether each statement is true. Select True or False for each statement.

	True	False
Parker's distance from home is increasing between minute 1 and minute 4.	☐	☐
Parker's distance from home is constant between minute 6 and minute 7.	☐	☐
Parker's distance from home is increasing between minute 12 and minute 14.	☐	☐

#4 Response Mode: Item Type

Come up with as many items types that you can based on what you know about the Smarter Balanced Assessment

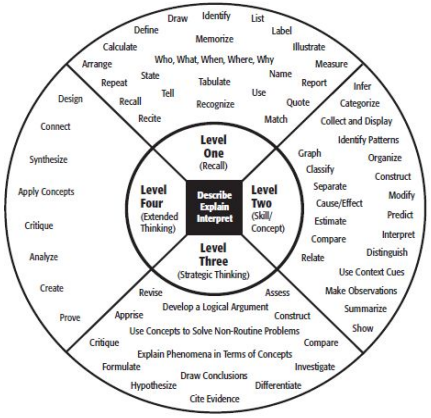
Content Area	Type of Item	Brief Description of How to Respond
ELA and Mathematics	Multiple Choice	select a single option
	Multiple Select	select two or more options
	Match Interaction	match text or images in rows to values in columns
	Short Answer Text Response	keyboard entry
Mathematics Only	Grid Item – Drag and Drop	drag-and-drop single or multiple elements into a background image
	Grid Item - Hot Spot	select certain areas of an image
	Grid Item - Graphing	plot points and/or draw lines
	Table Interaction	keyboard entry into table cells
	Equation Response	enter equation or numeric response using on-screen panel containing mathematical characters
ELA Only	Evidence Based Selected Response	two part item: Part A: Respond to a Multiple Choice item. Part B: Site the evidence that supports the answer to Part A either Multiple Choice, Multiple Select, or Hot Text
	Hot Text	select sections of text, or drag-and-drop sections of text
	Essay/Writing Extended Response	keyboard entry

#5 Processing Demand: DOK

Depth of knowledge is the interaction of how deeply a student needs to understand the content with different ways of responding and interacting with the content.

3 Minute Video:
https://vimeo.com/20998609

Depth of Knowledge (DOK) Levels



Level One Activities	Level Two Activities	Level Three Activities	Level Four Activities
Recall elements and details of story structure, such as sequence of events, character, plot and setting. Conduct basic mathematical calculations. Label locations on a map. Represent in words or diagrams a scientific concept or relationship. Perform routine procedures like measuring length or using punctuation marks correctly. Describe the features of a place or people.	Identify and summarize the major events in a narrative. Use context cues to identify the meaning of unfamiliar words. Solve routine multiple-step problems. Describe the cause/effect of a particular event. Identify patterns in events or behavior. Formulate a routine problem given data and conditions. Organize, represent and interpret data.	Support ideas with details and examples. Use voice appropriate to the purpose and audience. Identify research questions and design investigations for a scientific problem. Develop a scientific model for a complex situation. Determine the author's purpose and describe how it affects the interpretation of a reading selection. Apply a concept in other contexts.	Conduct a project that requires specifying a problem, designing and conducting an experiment, analyzing its data, and reporting results/ solutions. Apply mathematical model to illuminate a problem or situation. Analyze and synthesize information from multiple sources. Describe and illustrate how common themes are found across texts from different cultures. Design a mathematical model to inform and solve a practical or abstract situation.

Webb, Norman L. and others. "Web Alignment Tool" 24 July 2005. Wisconsin Center of Educational Research. University of Wisconsin-Madison. 2 Feb. 2006. <http://www.wcer.wisc.edu/WAT/index.aspx>

DOK Sort!

Depth of knowledge is the interaction of how deeply a student needs to understand the content with different ways of responding and interacting with the content.

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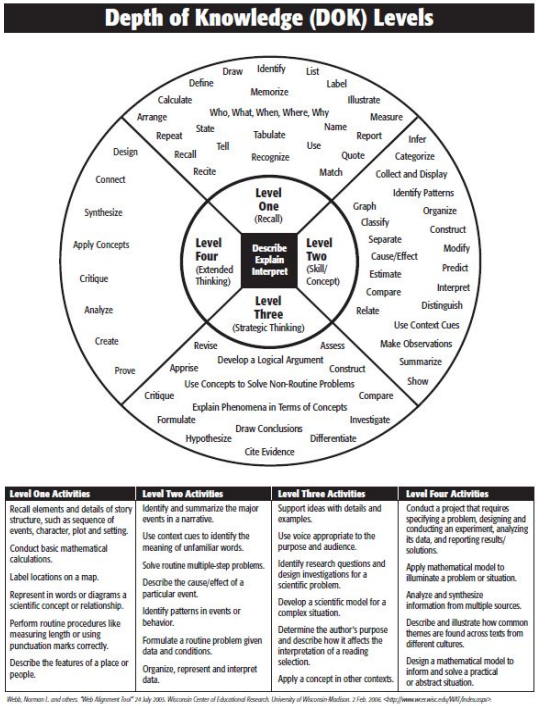

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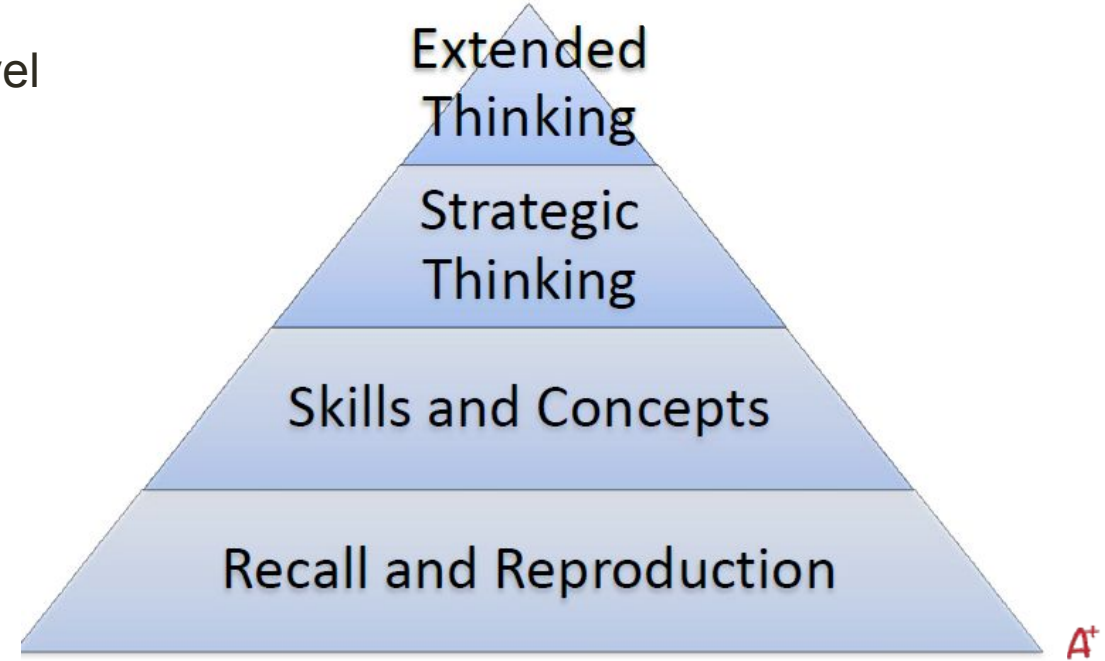
Webb, Norman L. and others. "Web Alignment Tool" 24 July 2005. Wisconsin Center of Educational Research. University of Wisconsin-Madison. 2 Feb. 2006. <http://www.wcer.wisc.edu/WAT/index.aspx>

DOK Sort

- Decide which DOK level each question meets.
- Discuss why each question is at the indicated level.
- Use Hess' Cognitive Rigor Matrix and DOK wheel to help you.



DOK Level 1: Recall

- Basic recall of concepts, definition, facts and processes
- Follow a simple formula
- Perform a routine procedure
- Simple skills and abilities

DOK Level 1: Math

Grade 3

Decide whether each expression is equal to 4×12. Select Yes or No for each expression.

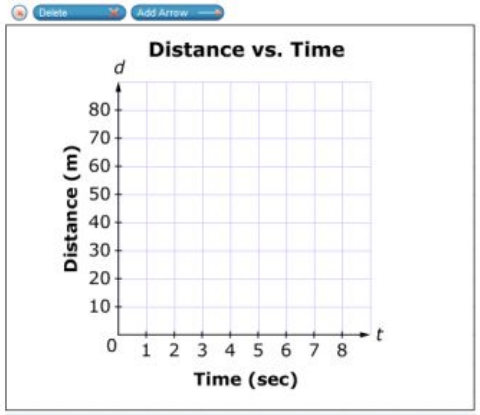
	Yes	No
$4 \times (10+2)$	☐	☐
$(4 \times 10)+2$	☐	☐
$4+(10 \times 2)$	☐	☐

Grade 8

The distance (d) in meters a car travels in t seconds is shown in the table.

d	t
10	1
20	2
30	3
40	4
50	5

Use the Add Arrow tool to graph the proportional relationship between the distance (d) traveled by a car and the time (t).



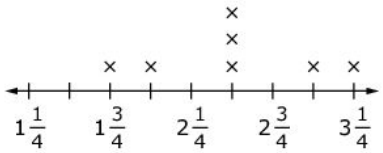
DOK Level 2: Basic Application

- Mental processing beyond recall
- Requires decisions as to how to approach the question or problem
- Perform two or more steps with decision points along the way
- Organize or display data, interpret or use simple graphs

DOK Level 2: Math

Grade 4

The line plot shows the distances, in miles, Rex walked on seven days.



Each × represents one day.

Distances Rex Walked (mi)

Enter the total distance, in miles, Rex walked for all seven days.

Grade 7

Alfonso went to Famous Sam's Appliance Store and purchased a refrigerator and a stove. The sale price of the refrigerator was 40% off the original price and the sale price of the stove was 20% off the original price.

Which statement must be true to conclude that Alfonso received a 30% overall discount on the refrigerator and the stove together?

(A) The sale prices of the refrigerator and the stove were the same.

(B) The original prices of the refrigerator and the stove were the same.

(C) The sale price of the refrigerator was twice the sale price of the stove.

(D) The original price of the refrigerator was twice the original price of the stove.

DOK Level 3: Strategic Thinking

- Reason or develop a plan to approach a problem
- Employ some decision-making and justification
- Solve abstract, complex, or non-routine problems
- Often allows more than one possible answer

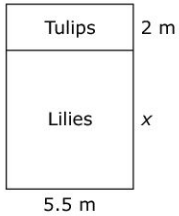
DOK Level 3: Math

Grade 6

3615

Jeremiah planted tulips and lilies in a field with a width of 5.5 meters. The field of flowers is shown.

Area of Flowers



- ☐ $A = x^2(5.5)$
- ☐ $A = 2x + 5.5$
- ☐ $A = 5.5x + 11$
- ☐ $A = 5.5(x + 2)$
- ☐ $A = x + 2 + 5.5$

Identify each equation that could be used to find the area, in square meters, of the field of flowers for any length x, in meters.

Grade 7

Mary and Jerry are exercising on a track.

- Mary is walking at a rate of 3 miles per hour.
- Jerry starts jogging at a rate of 4 miles per hour after Mary has been walking for 15 minutes.
- Jerry jogs 2 miles as Mary continues walking, and they both stop at the same time.

Enter the **total** distance, in miles, that Mary walks around the track.

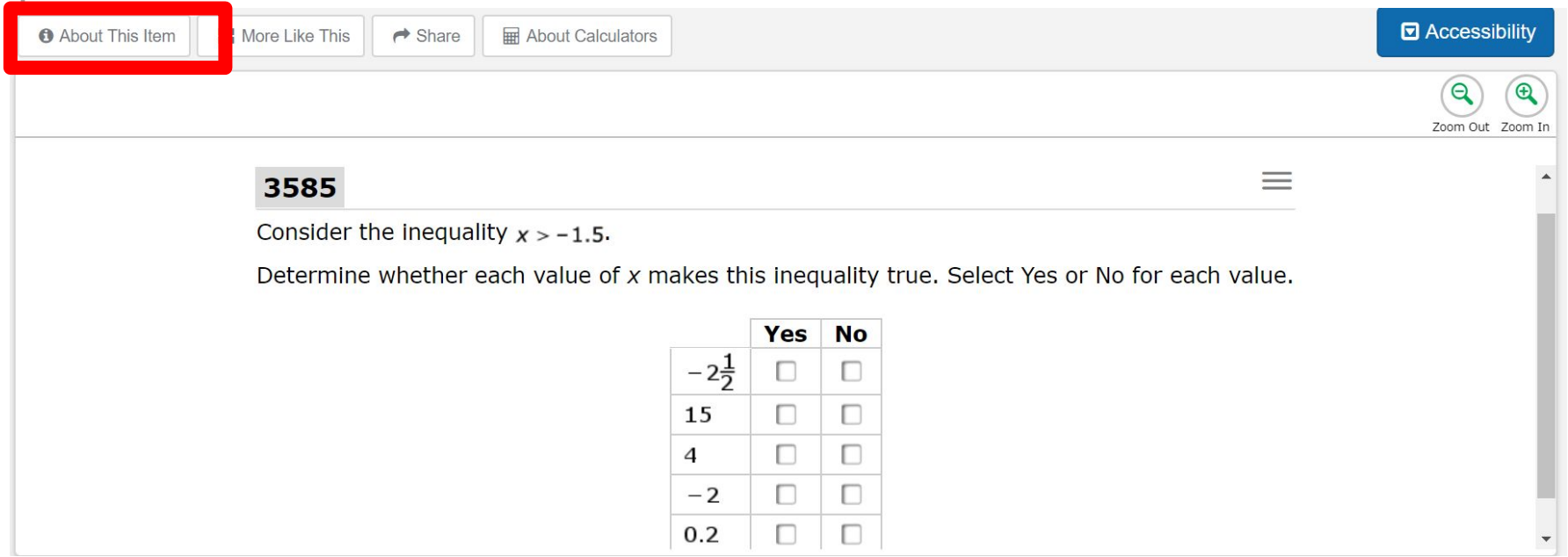
The Five Sources of Cognitive Complexity

1. Content: Claims + Targets + Standards
2. Practices: Scaffolding
3. Stimulus Material: Item Stimuli
4. Response Mode: Item Type
5. Processing Demand: Depth Of Knowledge (DOK)

Learner-Centered Design: A Cognitive View of Managing Complexity in Product, information and environmental design. .

By Wayne Reeves

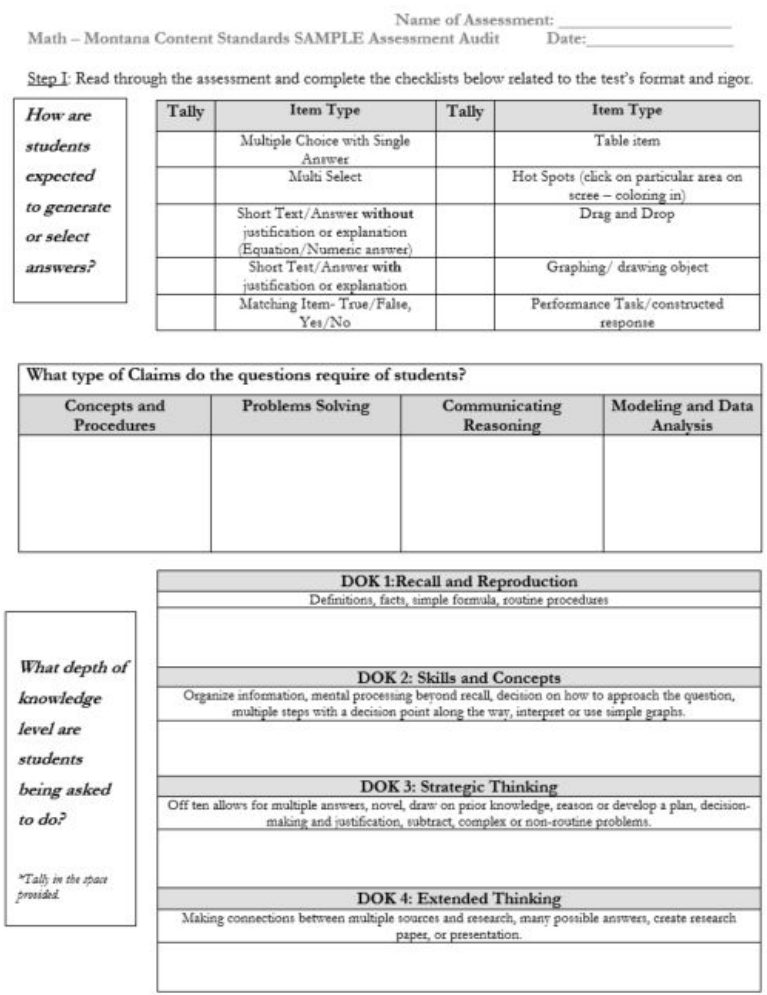
How do I create items that are rigorous?



www.sampleitems.smarterbalanced.org

Assessment Audit

- Individually
- Groups
- PLC
- Grade level
- Department level
- Administration



Name of Assessment: _____

Math – Montana Content Standards SAMPLE Assessment Audit Date: _____

Step I: Read through the assessment and complete the checklists below related to the test's format and rigor.

How are students expected to generate or select answers?

Tally	Item Type	Tally	Item Type
	Multiple Choice with Single Answer		Table item
	Multi Select		Hot Spots (click on particular area on scree – coloring in)
	Short Text/Answer **without** justification or explanation (Equation/Numeric answer)		Drag and Drop
	Short Text/Answer **with** justification or explanation		Graphing/ drawing object
	Matching Item- True/False, Yes/No		Performance Task/constructed response

What type of Claims do the questions require of students?

Concepts and Procedures	Problems Solving	Communicating Reasoning	Modeling and Data Analysis

What depth of knowledge level are students being asked to do?

**Tally in the space provided.*

DOK 1:Recall and Reproduction
Definitions, facts, simple formula, routine procedures
DOK 2: Skills and Concepts
Organize information, mental processing beyond recall, decision on how to approach the question, multiple steps with a decision point along the way, interpret or use simple graphs.
DOK 3: Strategic Thinking
Off ten allows for multiple answers, novel, draw on prior knowledge, reason or develop a plan, decision-making and justification, subtract, complex or non-routine problems.
DOK 4: Extended Thinking
Making connections between multiple sources and research, many possible answers, create research paper, or presentation.

Assessment Reflection

Name of Assessment: _____

Math – Montana Content Standards SAMPLE Assessment Audit Date: _____

Step II: Reflect on depth and balance of the math assessment by paying close attention to what students are being asked to do. Reflect on the use of text and inclusion of writing.

Discussion topics …
☐ The assessment directly addresses at least three mathematical practices ☐ The assessment is covering relevant and grade level standards ☐ The large majority of questions in are devoted to the Priority Clusters of the grade (Claim 1) ☐ Questions posed require students to demonstrate conceptual understanding of key mathematical concepts ☐ Questions explicitly assess procedural skill and fluency requirements ☐ Questions posed require students to show mathematical reasoning (Claim 3) ☐ Questions posed require students to show mathematical modeling (Claim 4) ☐ Item sequences do not cue students to use certain solution process during problem-solving (Claim 2) ☐ Problems require different types of solution processes throughout the assessment ☐ Items require a variety in what students produce ☐ A portion of the assessment includes writing that mirrors real-world activity (Claim 4)
Notes/Feedback

Strengths of Assessments	Areas for Improvement

Step IV: Make a plan.

Action Steps	Date

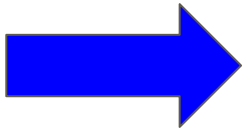
Upping the Rigor:Change DOK level

DOK1

Multiplying Fractions

Solve.

3/7 x 2/9 = ___



DOK2

Multiplying Fractions

Using the digits 1 to 9, at most one time each, fill in the blanks to make two different pairs of fractions that have a product of 2/3.

__ / __ x __ / __ = 2/3

*https://robertkaplinsky.com/3steps/

Upping the Rigor:Change DOK level

DOK 1

Multiplying Fractions

Solve.

3/7 x 2/9 = ___

DOK 3

Juan really likes his 6 times tables. He decides if he multiples 3/7 by 2/2 and then multiples that answer by 2/9 he will get the same answer as 3/7 x 2/9. Is Juan correct? What is the correct answer?

Original Question:

Multiplying Fractions

Solve.

3/7 x 2/9 = ___

Juan's work:

$\frac{3}{7} \cdot \frac{2}{2}$ then I will multiply that by $\frac{2}{9}$ and will get the same answer as the original problem.

*https://robertkaplinsky.com/3steps/

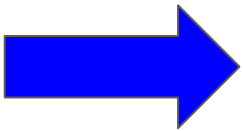
Upping the Rigor:Change Item Type

Multiple Choice:

Multiply or divide as indicated.

$\sqrt{-2} \cdot \sqrt{-2}$

A. $-2i$
B. 2
C. -2
D. $2i$



Matching Item:

Determine if the following product or quotient is an imaginary or real number solution.

	Imaginary	Real Number
$\sqrt{-2} \cdot \sqrt{-2}$		
$\sqrt{2} \cdot \sqrt{-2}$		
$\sqrt{-2} \div \sqrt{-2}$		

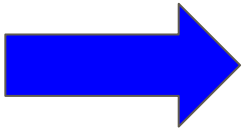
Upping the Rigor:Change the Claim

Concepts and Procedures

Alfonso went to Famous Sam's Appliance Store and purchased a refrigerator and a stove. The sale price of the refrigerator was 40% off the original price of the stove was 20% off the original price. How much money Alfonso save? The original prices are listed below:

Refrigerator: $850 Stove: $439

A. $427.80
B. $861.20
C. $345.60
D. $42.78



Communicating Reasoning

Alfonso went to Famous Sam's Appliance Store and purchased a refrigerator and a stove. The sale price of the refrigerator was 40% off the original price and the sale price of the stove was 20% off the original price.

Which statement must be true to conclude that Alfonso received a 30% overall discount on the refrigerator and the stove together?

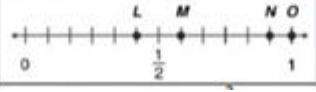
(A) The sale prices of the refrigerator and the stove were the same.

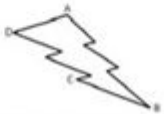
(B) The original prices of the refrigerator and the stove were the same.

(C) The sale price of the refrigerator was twice the sale price of the stove.

(D) The original price of the refrigerator was twice the original price of the stove.

Practice: Choose one item *DOK *Item Type *Claim

Topic	Adding Whole Numbers	Money	Fractions on a Number Line	Area and Perimeter	Subtracting Mixed Numbers
CCSS Standard(s)	• 1.NBT.4 • 2.NBT.5	• 2.MD.8	• 3.NF.2	• 3.MD.8 • 4.MD.3	• 5.NF.1
DOK 1 Example	Find the sum. $44 + 27 =$	If you have 2 dimes and 3 pennies, how many cents do you have?	Which point is located at $\frac{7}{12}$ below? 	Find the perimeter of a rectangle that measures 4 units by 8 units.	Find the difference. $5\frac{1}{2} - 4\frac{2}{3} =$

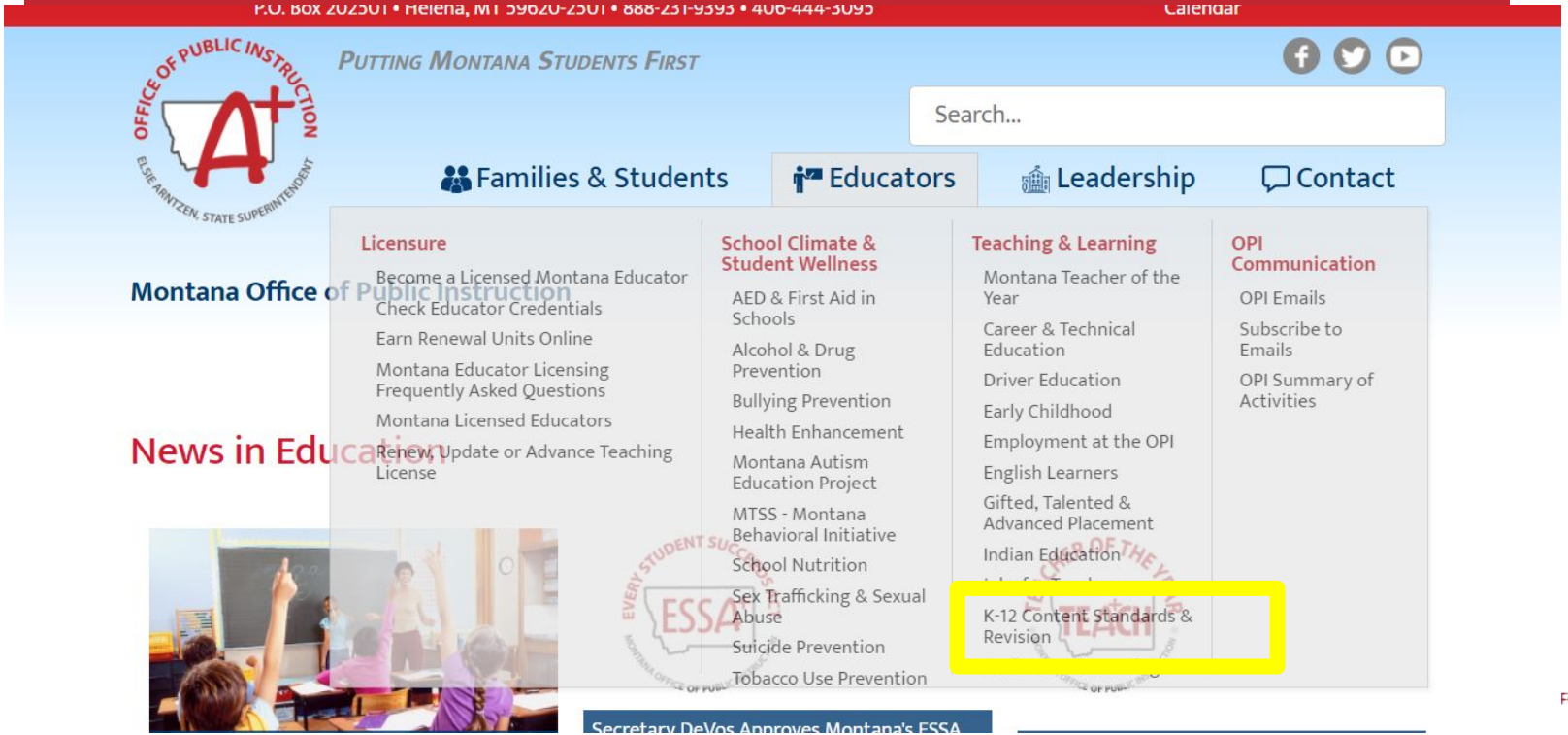
Topic	Surface Area and Volume	Probability	Transformations	Factoring Quadratics	Quadratics in Vertex Form
CCSS Standard(s)	• 6.G.4 • 7.G.6	• 7.SP.5 • 7.SP.7	• 8.G.1 • G-CO.5	• A-SSE.3a	• F-IF.7a
DOK 1 Example	Find the surface area of a rectangular prism that measures 3 units by 4 units by 5 units.	What is the probability of rolling a sum of 5 using two 6-sided dice?	Rotate the image below 90° counterclockwise about point D and reflect it across a horizontal line. 	Find the factors: $2x^2 + 7x + 3$	Find the roots and maximum of the quadratic equation below. $y = -3(x-4)^2 - 3$

Examples of increased DOK

Resources to Upping the Rigor

- SBAC Sample Questions
- DOK question Stems: Robert Kaplinsky + General Question stems
- DOK question cards
- Smarter Balanced Rubrics
- Problems of the Month
- SBAC Interim Assessments (3-8)

Where to find our Standards



OPI.MT.GOV

Now you have a rigorous assessment….

Leave you with:

"For the system to work…its elements must be aligned. That is, if an assessment is to communicate and measure expected standards for student performance, it must be fully aligned with those standards. Similarly, if classroom teaching and learning activities are to help students attain the standards, they too must be aligned with the standards. Finally, **it is only when assessment is aligned with both standards and classroom instruction that assessment results can provide sound information** about both how well students are doing and how well schools and teachers within them are doing in helping students to attain the standards." -Joan Herman and Noreen Webb, 2007

Set a Goal...

SIGN UP FOR UPCOMING MATH PD OPPORTUNITIES: MARISA.GRAYBILL@MT.GOV

September 2018	**October 7th- 28th** -Number Systems & Operations (K-3) -Number Systems & Operations (4-7)	**November 4th- 25th** -Ratios and Proportions (6-7) -Functions as Objects (HS)
December 2nd- 23rd -You Decide How to Divide (3-5) -Linear Relationships (7-9)	**January 6th 2019- 27th** -Developing Fraction Sense (3-5) -Transformations and Proofs (HS)	**February 3rd- 24th** -Algebraic Thinking (K-5) -Algebraic Thinking (6-7)
March 3rd- 24th -Measurement (K-3) -Describing Data (4-7) -Statistical Inferences (HS)	**April 7th – 28th** -Geometric Thinking (K-3) -Geometric Thinking (4-7) -Exploring Transformations (7-9)	**May 5th- 26th** -Connecting Length, Area, and Volume (K-5)
June 2nd- 23rd -Fraction Models and Operations (3-5) -Making Sense of Modeling (HS)	**July 7th – 28th** -Mathematical Practices (K-8) -Mathematical Practices (HS)	**August** **-Courses offered as requested**

STREAM- FREE 3 week Courses on the Teacher Learning Hub

-Enroll in courses today by at goo.gl/yho6DJ

-Find a full list of courses with descriptions at goo.gl/Rq3wjs

Montana Elementary Math Community!

-Monthly virtual gatherings

-Sharing of ideas and resources

- First meeting October 28 from 7-8pm

goo.gl/ra9D8i

Thank you!

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Session Materials:

goo.gl/GnfqwG

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